Proceedings of

The Fourth Bornö Symposium, June 1969

BORNÖ STATION

Preface

After a lull of three years the Bornö Symposium on geophysical hydrodynamics was again launched in the first week of June, 1969. This time the suggested topic for our scientific conversation was turbulence, and actually over half of the talks dealt with this subject. Other subjects dealt with were surface and internal waves, wind-driven and thermohaline ocean circulation and thermal circulation in laboratory experiments.

As usual one morning and one afternoon lecture were given each day, allowing ample time for the lecturer to develop his subject and for the audience to express disagreement. Some lectures actually lasted over four hours, leaving the audience exhausted but well enlightened on the scientific questions.

A few evening lectures were added to the program. George Veronis gave the "Ekman lecture" on stratified spin-up, in which he told us that the spin-up experiment was indeed well-known to V.W.Ekman already in 1906. Robert W. Stewart joined the Symposium and gave an evening lecture on ocean waves and Arne Foldvik did show films about some hydrodynamic laboratory experiments.

Several of the participants visited the University of Gothenburg before the Symposium, and gave lectures for the staff and students at the Oceanographic Institution. This time it was also possible to have some students come and listen to the Bornö lectures.

The weather was unusually cold and rainy this time, but the participants could see the sun during the boat trip up to Bornö as well as the last day when an excursion was made to Gåsö Island. The Symposium was formally ended in Lysekil where a short lecture on interacting rotating objects was followed by a Swedish Flag Day Dinner.

I would like to thank the participants at the Symposium for their scientific contributions which, I feel, made the Symposium very successful. We thank Mrs. Mary Thayer who kindly helped in correcting and typing our manuscripts, and Mr. and Mrs. Akermo, who took care of us so well during the stay at Bornö. Finally, we are grateful for the grant given by the Swedish Natural Science Research Council through contract no. 323-14, which made the Symposium possible.

Pierre Welander
Gothenburg, June 27, 1969.

Participants in the Symposium

Foldvik, Arne	Geofysisk Institutt, Universitet i Bergen, Allegatan 70, 5000 Bergen, Norway
Hasselmann, Klaus, Dr.	Institut für Schiffbau, Universitat Hamburg, Hamburg, Germany
Howard, Louis, Prof.	Institutionen, för Mekanik, Kungliga Tekniska Högskolan, Valhallavägen 79, Fack, 100 44 Stockholm 70.[1]
Landahl, Mårten, Prof.	Institutionen, för Mekanik, Kungliga Tekniska Högskolan, Valhallavägen 79, Fack, 100 44 Stockholm 70.[1]
Long, Robert, Prof.	Mechanics Department, The Johns Hopkins University, 34th and Charles Street, Baltimore, Maryland, USA
Mork, Martin, Dr.	Geofysisk Institutt, Universitet i Bergen, Allegatan 70, 5000 Bergen, Norway
Stewart, Robert, Dr.	Institute of Oceanography, University of British Columbia, Vancouver 8, Canada
Walin, Gosta	Internationella Meteorologiska Institutet, Tulegatan 41, 113 53 Stockholm
Welander, Pierre, Lab.	Oceanografiska Institutionen, Box 4038, 400 40 Göteborg 4
Veronis, George, Prof.	Internationella Meteorologiska Institutet, Tulegatan 41, 113 53 Stockholm.[2]
Wunsch, Carl	Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge.[3]

Scientific assistant at the meeting was Mrs. Mary Thayer, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts 02543, USA.

1) On leave from Mathematics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts

2) On leave from Department of Geology, Yale University, Connecticut.

3) On leave from Department of Meteorology and Oceanography, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Lectures given at the Symposium

June 2 Morning lecture by Robert Long:

Turbulence in Stratified Fluids

Afternoon lecture by Klaus Hasselmann:

Interpretation of Transport and Mixing Processes in the Ocean in Terms of Weak Turbulence .

June 3 Morning lecture by Louis Howard:

Upper Bounds in Turbulence Theory

Afternoon lecture by Mårten Landahl:

Statistical Analyses and the Ordered Features of the Turbulent Shear Flow Structure

June 4 Morning lecture by George Veronis:

Effect of Fluctuating Winds on Ocean Circulation

Afternoon lecture by Pierre Welander:

Ideal Fluid Regimes in the Ocean Circulation

June 5 Morning lecture by Martin Mork:

Quasi-stationary Wave Patterns at the Continental Slope

Afternoon lecture by Gösta Walin:

Contained Inhomogeneous Motion, or How to Produce a Stratified Fluid System

June 6 Morning lecture by Carl Wunsch:

Oceanic Internal Waves

Added evening lectures:

George Veronis: Stratified Spin Up (Ekman lecture)

Robert W. Stewart: Direct Measurements Relevant to Wave Generation

Arne Foldvik: Mountain Waves and Layering in Sugar-Salt Solutions (films)

Lectures given at the University of Gothenburg

May 30 George Veronis:

Distribution of Tracers in the World Ocean

Carl Wunsch:

Some Ocean Boundary Processes

Martin Mork:

Wind Action on the Sea

May 31 Louis Howard:

The Initial Value Problem for Rotating Stratified Flow

Mårten Landahl:

Statistical Analysis and the Ordered Features of Turbulent Shear Flow Structure

Robert Long:

Clear Air Turbulence

Abstracts of lectures at the Symposium

TURBULENCE IN STRATIFIED FLUIDS

by

Robert R. Long

Summary

An effort is made to understand turbulence in fluid systems like the oceans and atmosphere in which the Richardson number is generally large. Toward this end, a theory is developed for turbulent flow over a flat plate which is moved and cooled in such a way as to produce constant, finite fluxes of momentum and heat. The theory indicates that in a coordinate system fixed in the plate the mean velocity increases linearly with height z above a turbulent boundary layer, and the mean density decreases as z^3, so that the Richardson number is large far from the plate. Near the plate, the results reduce to those of Monin and Obukhov.

The curvature of the density profile is essential for maintaining the turbulent motion. When the curvature is negative as in flow over a flat plate, finite wave motion in a region lifts the fluid a certain distance upward. If this volume of fluid is subsequently mixed by wave-breaking, the turbulent patch flattens out at the new level well above the original center of mass, thereby transporting heat downward. A well-mixed volume of fluid will also tend to rise when the density profile is linear, but this rise is negligible on the basis of the Boussinesq approximation. The interchange of fluid of different, mean horizontal speeds in the formation of the turbulent patch transfers momentum. The mixing in the patch also destroys the mean velocity shear locally and transfers kinetic energy from mean motion to disturbed motion. The turbulence arises in spite of the high Richardson number because the waves in the interior tend to grow in resonance with the disturbances in the turbulent boundary layer.

This is possible because of the precise variations of mean density and mean velocity mentioned above.

The relationship between the curvature of the density profile and the transfer of heat suggests that the density gradient near the level of a point of inflection of the density curve (in general cases of stratified, shearing flow) will increase locally as time goes on. There will also be a tendency to increase the shear through the action of local wave stresses. We show that the net effect is a progressive reduction in Richardson number, and, therefore, an ultimate outbreak of Kelvin-Helmholtz instability. The resulting sporadic turbulence transfers heat (and momentum) through the level of the inflection point. This mechanism for the appearance of regions of low Richardson number is offered as a possible explanation for the formation of the surfaces of strong density and velocity differences observed in the oceans and atmosphere, and for the turbulence that appears on these surfaces.

INTERPRETATION OF TRANSPORT AND MIXING PROCESSES IN THE OCEAN IN TERMS OF WEAK TURBULENCE

by

Klaus Hasselmann

Stokes' theory of the mass transport induced by a sinusoidal gravity wave predicts a Lagrangian mean flow proportional to the square of the wave amplitude. The mass transport for a random wave field is given by a corresponding integral over the wave spectrum. The Eularian velocity field vanishes everywhere within the fluid, the Eularian mass transport being concentrated entirely in the surface layer. The solution applies to a non-rotating system. The effect of rotation basically alters the form of the mass transport. In the case of a homogeneous wave field, the mean Lagrangian current vanishes, whereas the mean Eularian current is equal to the negative non-rotational Stokes current. Superimposed on both Eularian and Lagrangian mean currents is a rotating inertial oscillation of frequency f. The amplitude of the oscillation is equal to the Stokes velocity. The modification of the Stokes solution follows from the fact that the Coriolis forces acting on the Lagrangian wave-induced currents cannot be balanced (in the case of a homogeneous wave field) by a horizontal pressure gradient. As there are no other horizontal forces acting on a Lagrangian flow, the velocity vector rotates with the local inertial frequency. In the Eularian reference frame, the additional negative Stokes flow is set up by the Reynolds stresses of the wave field, which in a rotating system are non-zero.

The solution remains a good first approximation also in the case that the wave field is slightly inhomogeneous. An expansion scheme describing the evolution of the mass transport by perturbing about the homogeneous solution can be developed in a general operator formalism. It is found that the current distribution spreads out in the vertical direction at a rate proportional to the horizontal Laplacian of the mean forcing wave field. In terms of a normal mode representation, the expansion scheme is valid if the forcing function is rich in high modes and low horizontal wave numbers, so that most of the mode frequencies are close to the inertial frequency. However, the normal mode decomposition need not be carried out explicitly, the sum of all modes of a given branch being treated as a single composite mode. In the present case, the composite modes do not satisfy the usual wave propagation relations, but a diffusion relation. The diffusion arises from the gradual phase mixing of a large ensemble of mode amplitudes which are originally generated in phase by an external force. The analysis applies in the opposite limit of the more usual geophysical situation, in which the modes are treated as statistically independent. Statistical independence is valid asymptotically; diffusion by phase mixing applies in the initial stages. In the present case, the initial stage includes many inertial oscillations.

The wave-driven mass transport may be relevant to the many observations of strong oscillatory currents with frequencies close to the inertial frequency. The currents are found at all depths and latitudes. The vertical scales are generally small compared with the depth, and the horizontal scales of the order of a few 100 km. They are strongly intermittent, with decay times of the order of 10 oscillations and are believed to show some correlation near the surface to the passage of storms. These features can be explained qualitatively by wave-driving mechanisms.

UPPER BOUNDS IN TURBULENCE THEORY

by

Louis N. Howard

In various problems of physical interest associated with turbulence we have external conditions which are time independent, but we know that the flow equations have many solutions satisfying these conditions, most of which depend on time. In such circumstances the (presumably unique) solution to the initial value problem no doubt depends in detail very sensitively on the initial data, but we expect that such details are not of primary physical interest, and that the quantities which are of interest are averages of various kinds, functionals of the flow variables, which have the same values for most of the solutions evolving from various initial conditions. Typically one cannot effectively describe the class of all solutions, and cannot compute such averages theoretically and it is then of interest to try to estimate them by upper (or lower) bounds; in favorable cases such upper bounds may be fairly close to values actually achieved by real flows, and at any rate they give some rigorous information (to the extent that they can themselves be rigorously obtained). Without an effective description of the class of solutions to the flow equations one cannot usually expect to find the upper bound for this class of functions, but by dropping some of the equations or replacing them by weaker forms (such as averages) one may be able to obtain a well-defined and tractable maximum problem whose solution will still give a reasonable upper bound. This idea was carried out to a certain extent for estimating the heat transport by turbulent convection between horizontal plates in my paper in J.F.M. 17, 405 (1963). In the present lecture some more recent developments along the same general lines and some improvements with regard both to simplicity and mathematical rigor are described.

In the 1963 paper the problem of maximizing heat transport subject only to the constraints of boundary conditions and the two so-called "power integrals" was formulated and the corresponding Euler equations solved, with the result that (asymptotically for large Rayleigh number R) the Nusselt number is bounded by $\frac{3}{8}\left(R/3\right)^{1/2}$. It turns out to be possible, by reasonably careful but elementary and completely rigorous estimates to obtain almost as good a bound without reference to the Euler equations; this was done with the result $N-1 \leq 2\left(R/3\right)^{1/2}$ in the 1963 paper, but in fact the result $N-1 \leq \frac{1}{2}\left(R/3\right)^{1/2}$ can be obtained.

In the 1963 paper the problem of finding an upper bound when the continiuty equation is added as a constraint was attacked, but it was solved only with the additional assumption that a single horizontal wave number was present, and only with a boundary layer method (and numerical calculation) for large R; the result $N \leq \left(\frac{R}{248}\right)^{3/8}$ was obtained. By direct rigorous estimates one can in fact obtain almost as much without any calculation; one finds

$$N-1 \leq \frac{5}{22}\left(\frac{3R}{11}\right)^{3/8} \cong 1.1\left(\frac{R}{248}\right)^{3/8}$$

Very interesting recent work by F. Busse shows that in fact the maximum is not attained with a single wave number, at least when R is large enough. Using boundary layer methods he finds bounds of the form $N \leq C_n R^{1/2(1-4^{-n})}$ when n wave numbers are allowed, with an overall bound $N \leq \left(\frac{R}{1035}\right)^{1/2}$. The structure is quite complicated, with n + 1 different boundary layer thicknesses, and it seems desirable to try to obtain rigorous direct estimates here as well, especially since there is some question about the validity of the boundary layer methods when infinitely many wave numbers are involved. This is in fact possible, and bounds of the same form as Busse's can be obtained, though the coefficients are not as close as in the case n = 1.

I have also found an overall bound for this case of inclusion of the continuity constraint without any reference to wave numbers; this is $N-1 \leq \frac{3}{16}\left(\frac{R}{3}\right)^{1/2}$. Although this result does not use the continuity constraint very strongly, it is nevertheless only about three times as large as Busse's value. These direct estimates would seem to suggest that Busse's boundary layer methods do indeed give the correct results and that questions about their mathematical rigor are not serious.

I have also made some (quite non-rigorous) investigations into the maximum heat transport problem for infinite Prandtl number when the full momentum equation rather than its power integral is used as a constraint. Here again a multiple boundary layer structure is found, with bounds of the form $N \leq C_n R^{\frac{1}{3}(1-10^{-n})}$. A detailed study of the boundary layers in the manner of Busse is now being completed by my student S. Chan. Although direct and rigorous estimates have not been found yet in this case, it appears that an overall bound of the form $N \leq C R^{1/3}$ applies here. Whether this improvement over the $R^{1/2}$ estimate is peculiar to the infinite Prandtl number case, or comes about because of the stronger constraint of the momentum equation is not known.

STATISTICAL ANALYSES AND THE ORDERED FEATURES OF
TURBULENT SHEAR FLOW STRUCTURE

by

Mårten Landahl

ABSTRACT

The problem of identifying ordered or recurring features in a turbulent shear flow from analysis of statistical measurements of the fluctuation field is discussed. The difficulty of this problem is illustrated by comparisons of statistical measurements with instantaneous flow pictures which often show very little relationship to each other. Simple model examples are given which demonstrate that the usual two-point covariances can easily overlook regular structures, particularly when intermittency is present. For intermittent fluctuations it may be advantageous to apply conditional sampling as utilized by Kovasznay and Kibens and by Laufer and Kaplan for studying the turbulent-laminar superlayer. For conditional sampling, averages are formed only during such times when turbulence occurs. A close inspection of the measurements of pressure fluctuations beneath a boundary layer reveals a structure describable as waves being excited by a distribution of sources. The waves are characterized by a dispersion relationship in good agreement with calculations based on the linear Orr-Sommerfeld equation (Landahl, J.F.M. 29, 3). Thus, the fluctuation field could be described as a linear "ringing" (in form of waves) caused by non-linear "banging" (the source term). This simple conceptual model is discussed in light of available experimental evidence. Wave number analysis of the pressure data obtained by Willmarth and Wooldridge shows that the source term has a spanwise structure related to the wave length of the linear wave. Also, the correlation length of the non-linear source is small compared to the

decay length of the linear wave. To demonstrate that this behaviour is consistent with the equations of motion one can write the equation for the fluctuating y-velocity component in the following way:

$$L(v) = g(x,y,z,t)$$

where L is the linear Orr-Sommerfeld operator

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\nabla^2 - U''\frac{\partial}{\partial x} - \nu\nabla^4$$

and g contains all the non-linear terms. The source function g consists of spatial derivatives of the fluctuating Reynolds stresses, the most important term being

$$-\frac{\partial^2}{\partial y^2}\frac{\partial}{\partial x}\left(\overline{uv} - uv\right)$$

Large contributions to g in the low wave number range would be possible when small scale fluctuations have a low wave number modulation, as would be the case when small scale fluctuations ride on the primary wave. Furthermore, the energy equation for parallel flow

$$\frac{\partial}{\partial t}\iint \frac{u^2+v^2}{2}\,dx\,dy = -U'\int \overline{uv}\,dy$$

- (viscous dissipation terms)

show that large correlations between the u and v components are associated with large growth rates of the fluctuations. Hence, the source term would be appreciable wherever there is a region of strong instability. Such regions can be produced by the wave-like fluctuations interacting with the spanwise streak structure to cause locally the appearance of inflexional velocity profiles which are known from inviscid stability theory to be highly unstable. These local secondary instabilities can be identified with the intermittent bursts in the wall layer observed in the experiments by Kim, Kline and Reynolds (Stanford Report MD-20). Thus, one would expect that the main source contribution would come from the bursts. This would also explain why the ciruclation scale of the source would be closely related to the wave length of the linear wave.

EFFECT OF FLUCTUATING WINDS ON OCEAN CIRCULATION

by

George Veronis

Three studies are reported for the response of a homogeneous ocean on the β-plane to a fluctuating wind-stress.

In the first study, the linear theory of N. A. Phillips, in which he attempted to relate the large amplitude eddy motions observed by Swallow and others to Rossby waves, is extended to take into account non-linear effects. Phillips' conclusions were that linear theory could correctly predict the frequency and wave length of the Swallow eddies but that the amplitude of the motions is too small by at least a factor of three. Phillips proposed that non-linear interactions of the Rossby waves with the "steady" Gulf Stream might result in an increase in amplitude of the predicted eddy motions. Numerical integration of the equations with non-linear processes included show that non-linear effects uniformly tend to reduce the amplitude of the oscillatory motions. This conclusion is obtained in the presence and the absence of a steady Gulf Stream. Hence, one is forced to expand the model to take into account baroclinic and/or topographic effects.

The second calculation is made in order to determine the mean circulation which can be generated by fluctuating winds. A time harmonic wind-stress is applied to the ocean surface from t = 0 and the equations are integrated until the system settles down to a cyclic behavior. The solution at each (spatial) point is then integrated over two cycles and the mean circulation is thus derived. It is found that the mean response of the system is non-zero, i.e., a mean circulation is derived and the amplitude of the response is a function of the driving frequency. For high and low frequencies the mean response is low and for intermediate frequencies the mean response is maximum. The reason for the dependence is explained by the combined effect of local and non-linear processes relative to the β-effect. Where the β-effect does not have time to become fully established (high frequency forcing) the mean response, which is dependent on β, is small. When the frequency is low, the time response is quasi-static and the transient motions do not interact with each other sufficiently to produce a mean circulation. The maximum amplitude of the mean response, measured in terms of Gulf Stream transport, is about half that which is produced by a steady wind of the same amplitude.

In the third study a time harmonic wind-stress is superimposed on a steady wind-stress and the mean circulation is then deduced by averaging the transport at each location over ten cycles of the fluctuating wind. The latter averaging process yields a mean circulation which fluctuates by less than 2% over the tenth cycle of oscillation. Hence, it can be used as an accurate measure of the mean circulation. The results show that if the flow due to the steady wind-stress acting alone is strongly non-linear and if the fluctuating wind-stress has twice the amplitude of the steady stress, the mean transport is increased by a factor of two over the circulation due to the steady stress acting alone. Hence, the conclusion is that fluctuating winds can contribute significantly to the mean transport of the ocean. A single calculation shows that in order to deduce the same transport from the mean wind-stress acting alone it would be necessary to double the strength of the latter. Hence, in wind-driven ocean circulation theories it appears that neglect of the transient winds leads to a serious underestimate of the resulting circulation.

QUASI-STATIONARY WAVE PATTERNS AT THE CONTINENTAL SLOPE

by

Martin Mork

Temperature and salinity sections in the Norwegian Sea taken normal to the eastern continental slope reveal a waviness of the isotherms and isohalines. The waves are apparently stationary during the time of observation, but may be of long periodic nature. The wavelengths, which seem to increase towards the continental slope, are of order 50 km. Another interesting feature of these waves is the lack of phase shift with depth and the maximum amplitude at the pycnocline. This indicates that a two-layer model may be appropriate for a theoretical discussion. One expects that the pattern is related in some way to the bottom topography and the Norwegian Atlantic Current. A model is advanced which shows that the perturbation of a basic current in terms of long periodic waves propagating in the current direction provide a possible explanation. The slope of the interface and/or the bottom are essential for the existence of long periodic standing wave patterns in the direction perpendicular to the direction of propagation.

One of the simplest cases to solve for is that of a wedge-shaped upper layer and a deep lower layer with compensating motion. Assuming that the period is much larger than the inertial period, the differential equation which determines the x-dependency (x-direction normal to the coast) of the vertical displacement of the interface is

$$\frac{\partial}{\partial x}\left(h\,\frac{\partial \xi}{\partial x}\right) + R^{-1}\,\frac{c}{c-V}\,\xi = 0,$$

where $R = \frac{V}{fL} \sim 10^{-2}$ is the Rossby number given by the width of the wedge, L, and the basic current, $h = h_o + x$ is the non-dimensional depth of the upper layer, x = x/L, and c, the phase velocity, is an eigen-value parameter to be determined by the homogeneous boundary conditions. The solution is given by Bessel-functions of zero order and of argument $q = 2\left(\frac{h}{R}\,\frac{c}{c-V}\right)^{1/2}$. Thus

$$\zeta = \sum_n A_n Z_0 \left(q_n(x)\right) e^{ik(y-c_n t)} .$$

The first eigen-value solution gives $c_1 \approx \frac{V}{40}$, but as the eigen-values are increased, c_n tends towards V. In a case where the wavelength in direction of propagation is of order 100 km, the corresponding periods lie between one year and a week. Accordingly a layer with sloping interface may act as a resonator to long periodic disturbances. The general picture obtained from this model fits the observed wave pattern well, but it remains to be tested if there is any phase change with time in x-direction as predicted. The effect of the bottom topography and the coupling between the two layers also need to be investigated.

DIRECT MEASUREMENTS RELEVANT TO WAVE GENERATION

by

Robert W. Stewart

Measurements by Snyder and Cox (1966) and by Barnett and Wilkerson (1967) of the growth of ocean waves in fetch-limited situations have indicated the existence of a range of wave frequencies, lower than the spectral peak, for which energy grows exponentially with fetch. The observed wave growth, however, is several times greater than is predicted by the only well-developed theory of exponential wave growth - that of Miles (1957). There exist, at the same time, theories due to Hasselmann (1966), to Benjamin and Fier (1966) and to Longuet-Higgins (unpublished) which lead to transfer of energy to lower from higher frequency waves.

Of these, the Hasselmann theory is to some extent supported by the observations of Barnett and Wilkerson (1967) and by Hasselmann (unpublished) which show that for any particular frequency in a fetch-limited situation the growth is such that energy increases roughly exponentially until the energy at that frequency forms the peak of the spectrum, but at longer fetches where still lower frequencies become dominant, it decreases significantly.

Hasselmann's theory calls for non-linear interactions to transfer energy from waves rather higher in frequency than the spectral peak to the rapidly growing waves with frequencies just lower than the peak. This is more directly in line with these observations than is the Benjamin and Fier theory, which calls for non-linear transfers among closely neighboring frequencies, or the Longuet-Higgins "maser" mechanism which calls for waves of quite high frequency to be the source of low-frequency wave energy.

In view of the existence of these non-linear mechanisms, it is not clear how much of the energy going into a given wave frequency is transferred directly from the air, and how much first goes through some other frequency. Further, the relative importance of wave generation to the whole air-sea interaction process is not entirely clear.

For these reasons Dobson (unpublished) in Vancouver undertook an experiment to determine _in situ_ ($\sim$ 20 cm diameter) buoy containing a pressure sensor on its upper surface - carefully located with the aid of wind tunnel tests to minimize effects produced by the buoy. This buoy was mounted in such a way that it was free to rise and fall with the waves, but was horizontally constrained by running a capacitance wave probe vertically through a gimballed sleeve in its centre. The pressure sensor was made capable of measuring pressure differences appreciably less than one microbar.

Pressure and wave height signals were recorded on an analogue tape recorder, as was the wind speed at a few meters height, and sometimes the three turbulent wind velocity components.

Co-spectra and quadrature-spectra for wave height and pressure signals were computed. From this it is possible to calculate the momentum transfer - which is given by the pressure with wave slope covariance - and the energy transfer which is given by the pressure with vertical velocity covariance.

The results showed that within experimental error ($\sim$ 20%) _all_ of the momentum transfer from the air to the water is in the form of wave generation. Phase angles between waves and pressure were usually in the neighborhood of 90°, much higher than predicted by Miles and very efficient for wave generation.

The momentum transfer was distributed spectrally in such a way that the Snyder and Cox empirical formula

$$\frac{\dot{E}(k)}{E(k)} = k \, \frac{\rho a}{\rho w} \left\{ U(k) - C(k) \right\}$$

was well obeyed over a wide frequency band. Here E(k) is the energy flux density into waves of wave number k and energy density (E(k), $C(k) = \sqrt{g/k}$, the phase celerity, and U(k) is the wind speed at some suitable height, taken by Snyder and Cox as one wave length but by Dobson as k^{-1}.

These observations were made in a severely fetch-limited situation, with fetches of only a few kilometers so that the waves generally were very far from "saturated". They do not indicate that any non-linear wave-wave mechanisms are required to account for observed wave growth. (However in less fetch-limited cases, where the factor $U - C$ tends to be smaller, this may cease to be the case.)

In situations where $U - C$ changes sign, there was definite evidence of reverse momentum transfer - i.e., from waves to air. This is a wave-damping mechanism, perhaps related to the common observation that waves running against the wind tend to lose energy rather rapidly, at least when they are moderately steep.

CONTAINED INHOMOGENEOUS FLUID MOTION,
OR HOW TO PRODUCE A STRATIFIED SYSTEM

by

Gösta Walin

An effort is made to predict the basic density distribution and the lowest order motion in a closed container subject to thermal driving at the boundaries. The analysis is limited to the parameter range in which the density in the main part of the container is essentially a gravitationally stable function of z and t. The problem is attacked with boundary layer methods. It is found that the equations governing the boundary layer part of the fields become linear if the density variation, when passing through the boundary layer, is small compared with the scale of the interior variation. This condition may be met either by linearizing around a known basic density distribution, in which case we must give up our task to make a non-trivial prediction of the basic density field, or by imposing a condition on the boundary condition, which is supposed to be of the general linear form

$$\frac{\partial}{\partial n}\rho = S\left(\rho-\rho^{s}\right),$$

namely that S should satisfy the fairly weak constraint

$$S << \delta^{-1}$$

where δ is the thickness of the boundary layer. Under this restriction the vertical transport in the boundary layer may be expressed in terms of the local interior fields and the local boundary condition. Integrating this expression at each level we get the total vertical boundary layer transport which in turn gives us the vertical distribution of vertical velocity in the interior. This is precisely what we need in order to close the equation governing the interior density distribution.

The resulting equation turns out to be linear in ρ^I despite the fact that we still are able to predict a non-trivial basic distribution of density.

The theoretical results are used to discuss possible ways to produce and maintain a stratified fluid system. By proper construction of the "side-walls" (or rather "non-horizontal boundaries") any desired distribution $\rho^I(z)$ may be obtained as a true steady state making use of two constant-temperature baths to drive the system. Of particular importance is the fact that the time constant of the system may be controlled to some extent. For example, it may be made reasonably small, since we are not limited by the internal diffusion time. These ideas were also illustrated with a primitive experimental set up, showing how the thin boundary layers control the interior distribution of density.

IDEAL FLUID REGIMES IN THE OCEAN CIRCULATION

by

Pierre Welander

It has generally been believed that the main thermocline in the oceans represents a thermal boundary layer, in which vertical advection upward of cold deep water and vertical diffusion downward by heat enter critically. With a vertical velocity w and a vertical eddy conductivity K one gets a thermocline thickness $\delta \sim \frac{K}{w}$. Values $K \sim 1\ cm^2 s^{-1}$ and $w \sim 10^{-5} cm\ s^{-1}$ have been considered typical, giving $\delta \sim$ 1 km. There are, however, certain reasons to believe that the main thermocline may be an ideal fluid regime, and that the real thermal boundary is much thinner, comparable to the Ekman layer:

1. T-S diagrams taken from vertical stations in many oceanic areas agree well with the T-S diagram taken along a poleward section in the surface water during winter (Sverdrup et al "The Oceans"). Even non-linear features in the T-S diagram can be seen preserved, suggesting an ideal fluid sinking of surface water along density isosurfaces.

2. The value $K \sim 1\ cm^2 s^{-1}$ is not measured directly, but deduced from a diffusive-type model. Some direct measurements were reported on by Robert Stewart during the meeting, and point to much lower values, only a few percent of the above.

3. Solutions for the density field can be obtained from the geostrophic ideal fluid equations which agree very well with the observed density distribution in the thermocline. The derivation goes as follows:

For steady, geostrophic and ideal flow, both potential vorticity (P), Bernoulli function (B) and density (ρ) are conserved along streamlines. Hence a functional relation $P = F(B, \rho)$ must exist. It is assumed that the earth has a radial gravity field, and that the oceanic vertical scale is small compared to earth's radius and to the horizontal scale. One has $P = 2\Omega \sin\varphi / \frac{\partial \rho}{\partial z}$, $B = p + \rho g z$, where z is a vertical coordinate and φ the latitude. In the vertical direction one can deduce a hydrostatic balance. Hence one finds the two equations

$$\sin\varphi / \frac{\partial \rho}{\partial z} = F\left(p + \rho g z, \rho\right) \tag{1}$$

$$\frac{\partial p}{\partial z} = -g\rho \tag{2}$$

that give ρ and p (2Ω is absorbed in F). A simple case is obtained when F is linear: $a(p + \rho g z) + b\rho + c$. Taking a z-derivative of (1) and using (2) gives

$$\frac{\partial^2 \rho}{\partial z^2} = (agz + b)\frac{\partial \rho}{\partial z}$$

which has the solution

$$\rho = \rho_1(\lambda,\varphi) + A(\lambda,\varphi)\int_0^z e^{-\left(\frac{z'+z_0}{D}\right)^2} dz'$$

where $\rho_1(\lambda,\varphi)$ and $A(\lambda,\varphi)$ are arbitrary, z_0 is a constant giving the depth to the inflexion point, and D is proportional to $(\sin\varphi)^{1/2}$, representing a thermocline width. All the main features of the thermocline along a mid-oceanic meridional section are well reproduced by this solution. Note that a two-layer structure is obtained just at the equator. A numerical comparison has been made for the Pacific, adjusting ρ_1 to observed surface densities, A to a bottom density, and determining z_0 and D to fit a profile as well as possible at 30°. A calculation shows that the velocity is downward, at least in the subtropical gyre. Thus a reversal of the vertical velocities downward produced in the Ekman layer is not needed in this model, as it is in the diffusive type models. The fate of the sinking water is not clear. Somewhere it must pass through a diffusive region and it is suggested that such regions are found near the equator and at the coasts. A more complete calculation with prescribed density and Ekman vertical velocity at the top is being planned.

It is pointed out that a bottom condition $w = 0$ at $z = -H$ only can be satisfied by keeping $F(B, p)$ arbitrary. The analytical solution given above is too restrictive. The same difficulty occurs in the analytic thermocline models with diffusion. Here the restriction is imposed by using similarity solutions.

OCEANIC INTERNAL WAVES

by

Carl Wunsch

The main features of oceanic internal waves that need to be properly measured are their absolute energy content, the distribution of energy as a function of frequency and wave number, their direction of propagation and the sources driving the waves. In order to start measuring these quantities, an "antenna" has been constructed in the main thermocline at Bermuda.

The antenna is a cable stretched out normal to the bottom contours of the island with three thermistors 1 km apart horizontally at a depth of 700 m. The depth of water is from 1000 to 2000 m. The cable is maintained in a horizontal position by 1800 8" fishing floats. The power spectra of the temperature oscillations are all monotonely increasing with period. They are essentially flat at frequencies above the local Brunt-Väisälä frequency with a slight decay toward the highest + frequencies measured (period of 28 seconds). The energy density appears to be higher in shallow water at all periods so far measured (1 week to 28 seconds).

A small internal semi-diurnal tidal peak of 0(3-4 m) double amplitude can be resolved as a sharp line. There is no sign of a significant diurnal tide. Comparatively strong tidal harmonics exist at the shallowest sensor.

The coherence of the sensors is discouragingly low even at long periods. The array of sensors should operate as an antenna for internal wave length of the order of the sensor spacing. For mode 1 this corresponds to a period of about 20 minutes. Unfortunately there is no usable coherence and hence no chance to obtain directional spectra.

The simplest theory of internal waves on a sloping bottom shows that wave energy propagating up a slope should be absorbed at the apex for periods shorter than a critical period which depends upon the local bottom slope and Brunt-Väisälä periods. Above this period partial reflection of the wave energy should occur. On this basis the energy density for short period internal waves should increase shoreward and for long period internal waves should decrease. The results, as mentioned, show an increase shoreward at all frequencies.

Both the lack of coherence and the anomalous energy increase can be explained away by assuming that the temperature fluctuations are due to an entirely turbulent field, or to the advection past the sensors of "frozen turbulence". A more appealing explanation can be found by examining a series of STD sections made around Bermuda between the 200 and 2000 m isobaths in a series of spokes. The sections indicate a very marked systematic increase in the mean square temperature gradient as the island is approached. Large (50 m thick) steps appear close to the island with transition regions a few meters thick. A simple calculation shows that a thermistor sinusoidally oscillating through a temperature discontinuity will generate a spectrum of temperature decaying away toward high frequencies as $1/w^2$. This is at least of greater amplitude than the observed spectra. Thus in shallow water, the large amplitude low frequency oscillations will tend to induce spurious high frequency energy in the temperature spectra which will both increase the apparent energy and destroy the coherence with neighboring sensors. The remedy is to measure temperature gradients as well as temperatures. The presence of the steps in temperature near the island is associated with changes in the mean properties of the water column. Near the island the surface water is slightly heavier than far away. This can be expressed either in terms of dynamic height or potential energy anomaly. Deeper down (below ~ 600 m) the reverse seems to be true. This is consistent with a picture of mixing taking place through, for example, the breaking of internal waves. Such an effect might also be consistent with a bending of the density surfaces by the radiation stress associated with incoming internal waves. A calculation of the effect shows that it is probably too small to account for the observed values of potential energy anomaly.

Some help in both questions of a source for internal waves and the coherence problem can be found in an experimental program of study of the non-linear interactions of the waves. In a study of resonant trio interactions in a stratified fluid 21 m long, 1 m deep and 1½ m wide, it was found that the process of feeding energy into modes not directly generated was extremely rapid with multiple resonant triads possible. This enormously complicates the problem of tracking internal waves over the ocean, but it may simplify the source problem. For example, the internal tide may be able to provide energy over the entire range of observed frequencies. What is required is an estimate of the transfer rate and the rate of decay of the oceanic internal wave spectrum.

STRATIFIED SPIN-UP
(Ekman lecture)

by

George Veronis

Consider a homogeneous fluid contained in a cylindrical container rotating uniformly with angular rotation rate, Ω_o , about the axis of the cylinder. If the rotation rate of the cylinder is changed to a new value, $\Omega = \Omega_o + \Delta\Omega$, the fluid must adjust to the new rotation rate. The flow by which this adjustment takes place when the system is linear $(\Delta\Omega << \Omega_o)$ has been analyzed by Greenspan and Howard, who have given the name "spin-up" to the process. Recent articles by Holton, Pedlosky and Walin have served to clarify the spin-up process for a stratified fluid.

The original spin-up experiments for both a homogeneous and a stratified fluid were actually carried out by Ekman in 1906. Ekman was specifically interested in the vertical transport of momentum from the bottom boundary upward into the fluid. To study the latter process he did the spin-up experiment with $\Omega_o = 0$ with sand particles attached to the bottom boundary. The purpose of the sand was to roughen the bottom boundary, which, Ekman supposed, would then be the dominant boundary for transferring momentum to the fluid. He made use of the solution to the ordinary diffusion equation with an eddy viscosity coefficient. Measuring the rate at which rotation was transmitted to the mid-layers of the fluid, he used the solution to determine the value of the eddy coefficient. He concluded that in a layered fluid stratification cut down the eddy transfer coefficient.

Pettersson (1930) redid Ekman's experiments but with a continuous rather than a layered stratification and came to the same conclusions as Ekman. However, he also made additional observations and found that the rotation of the boundary was transmitted to the fluid via an indirect circulation of the entire fluid, i.e., he observed the spin-up flow. He concluded from this that the experiment was not suitable for studying the vertical transfer of momentum into the fluid by turbulent process and subsequently carried out another set of experiments more appropriately oriented toward the original purpose. Thus Ekman and Pettersson actually did the spin-up experiments and Pettersson, in particular, reported numerous observations which showed that he had essentially observed the spin-up flow. Because they were both oriented toward studying turbulent momentum processes, they did not pursue the study of the observed spin-up flow.

Recent measurements of the spin-up of a stratified fluid carried out by George Buzyna and me at Yale University show that Walin's theory correctly describes the larger scale processes in the response of the fluid. The agreement between theory and experiment is within the limits of experimental error for measurements made in the central regions of the fluid. Closer to the boundaries, where the spin-up process is most effective, Walin's theory underestimates the spin-up by 15-30%. The reason for this can be understood by studying the results of two simple theoretical models. The first shows the spin-up response in a two-dimensional fluid to spatially harmonic forcing at the top and botton boundaries. From this simple model it is directly seen that large-scale boundary effects penetrate deep into the fluid and small-scale effects are limited to regions closer to the boundary. A second model with both large- and small-scale forcing at the boundary shows the difference in response to the different scales. Hence, the conclusion is that Walin's theory correctly predicts the large-scale response but fails to predict the smaller-scale response adequately.

Corrections to Walin's theory using linear processes which are not included in his analysis improve the agreement near the outer boundary of the cylinder but fail to improve the theory near the bottom and top. It appears to be necessary to take into account the non-linear behavior near the corner regions in order to improve the agreement.